ESTIMATION OF THE INTENSITY OF LONGITUDINAL MIXING IN A FINITE CHANNEL

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A new approach is proposed and an expression is obtained for the estimation of the intensity of longitudinal mixing in a channel of finite length.

The following principal drawbacks are inherent to the method of unweighted statistical moments, which is widely used for the estimation of time-invariant parameters of models of flowing systems.

1. The accuracy of the calculation depends on the presence and configuration of the descending branch of the signal (the "tail"), where the current values of a concentration perturbation slowly approach the asymptote with time, which is characteristic of the subjects of chemical processing. The role of the "tail" as a source of errors grows considerably with an increase in the order of the moment being calculated. Therefore, only the first and second statistical moments are used in the determination of the parameters of the model, i.e., a maximum of two parameters are estimated.

2. The calculation of the moments (usually through the discrete representation of a continuous signal) occupies a considerable time. Since the effect of the time factor is not compensated for by a weight factor, the use of quadratures of the Gaussian type with nonequidistant nodes [1] is ineffective in the calculation of the moments.

3. The volume of calculations increases sharply in the estimation of the mixing parameters in a finite channel, especially in the case of an impulse perturbation of arbitrary form [2].

4. When the first two moments are used it is impossible to test the suitability of the adopted model; the latter is assumed a priori to be adequate [2].

The absolute contribution of the "tail" to the calculated moment is sometimes estimated by postulating [3, 4] an exponential character for the damping of the response curves, which is justified [5] for open channels. However, this assumption does not affect a portion of this least accurate part of the signal.

In the calculation of the k-th moment one can reduce the degree of the time factor by unity if instead of an impulse perturbation one imposes on the system a stepped signal or a perturbation growing with a constant rate [6]; in this case the first two moments represent quadratures or an initial ordinate.

The estimation of the parameters of a model, especially for open and semiconfined channels [7-13], is considerably simplified by the method of harmonic analysis and the synthesis of signals of arbitrary form [14-16] with its modifications [17, 18], and by the method of weighted moments [7, 8, 19]. These methods allow one to accomplish the controlled reduction of the influence of the "tail;" moreover, with their use (especially the former) the zeroth moment carries a greater weight of meaning. Here also, however, the choice of the upper limit of the working band of frequencies in the harmonic analysis of the signals and of the scale factor in the method of weighted moments remains very uncertain, which can lead to considerable errors in the values of the parameters being determined. These two methods, as well as the method of least squares using a fast Fourier transform with minimization of the discrepancy between the impulse function of the system and the model [20], which is close to them, help to test the adequacy of the model description.

The ideas of the method of weighted moments and the analysis–synthesis concept come together in the Laguerre method of orthogonal functions [21], the efficiency of which was demonstrated in a representation of a flowing system by a set of vessels for total mixing [19, 21]. In the present report this method is applied to the problem of the identification of flowing systems in which the flow corresponds adequately to a diffusional model

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$$\frac{\partial c}{\partial t}=\frac{1}{P}\frac{\partial^2 c}{\partial x^2}-\frac{\partial c}{\partial x} \tag{1a}$$

with the boundary conditions [22]

$$\lim_{t\to 0} c(x,\ t)=0,\quad x\in(0,\ 1]; \tag{1b}$$

$$c(0,\ t)=y_1(t),\quad y_1(t)\in D'_+; \tag{1c}$$

$$\frac{\partial c}{\partial x}(1,\ t)=0, \tag{1d}$$

where $P = uL/E_L$ is the Peclet number; L is the length of the working section of the channel; u is the velocity of the liquid in it; E_L is the coefficient of longitudinal diffusion; $c = C/C_0$ is the dimensionless concentration, equal to the ratio of the true concentration C to the reference concentration C_0; $x = z/L$ and $t = \tau/\Theta$ are the dimensionless coordinate and dimensionless time; z is the current coordinate; $\Theta = L/u$ is the average time of stay; and $y_1(t)$ is the concentration perturbation, taken from a subspace of distributions D'_+ of finite order [23].

The problem was solved [22] using a Laplace transform, and therefore it is convenient to determine the transfer function (the e curve) of the boundary problem (1) at x = 1 from the displacement theorem [23]

$$e(t)=e(1,\ t)\circ\!\!-\!\!\bullet E(1,\ s)=\alpha K(s), \tag{2}$$

where

$$\alpha=\exp(P/2), \tag{2a}$$

$$\varkappa(t)\circ\!\!-\!\!\bullet K(s)=\frac{\sqrt{P\left(s+\frac{P}{4}\right)}}{\sqrt{P\left(s+\frac{P}{4}\right)}\operatorname{ch}\sqrt{P\left(s+\frac{P}{4}\right)}+\frac{P}{2}\operatorname{sh}\sqrt{P\left(s+\frac{P}{4}\right)}}. \tag{2b}$$

The method of representation* of the experimental impulse function $\tilde{\varepsilon}(t_1)$ on the basis of the recorded curves $y_1(t)$ and $Y_2(t_1)$ of the input and output signals through series expansions by N orthogonal Laguerre functions $\Psi_i(t)$ and $\Psi_i(t_1)$, respectively ($i = 0, 1, \ldots, N-1$), has been demonstrated on the examples of various dynamic subjects [21], in particular, in the analysis of the concentration perturbation–response curves for liquid fluidized systems [24]. As in the harmonic analysis of dynamic systems based on the data of the perturbation by a single impulse and the reaction to it [25], for the more precise representation of the output signals in the form of Laguerre series their time reference points are shifted to a point preceding the first departure of these signals from the background level (the null):

$$t_1=t-d_0, \tag{3}$$

where $d_0 = \tau_d/\Theta$ is the relative delay of the signal; τ_d is the true time delay or "dead" time.

With a fixed value of the time scale factor $a > 0$ the Laguerre functions of the zeroth and first orders have the form

$$\Psi_0(t)=\sqrt{a}\exp\left(-a\frac{t}{2}\right),$$
$$\Psi_1(t)=\sqrt{a}\exp\left(-a\frac{t}{2}\right)(1-at). \tag{4}$$

The coefficients $\tilde{P}_i$ and $\tilde{q}_i$ to the Laguerre functions are expressed through the improper integrals $\int_0^\infty y(t)\Psi_i(t)dt$.

As yet there are no clear recommendations in the literature concerning the choice of the scale a and of the highest order $N - 1$ of the Laguerre functions. To fill in this gap in terms of the boundary problem (1) we determine the coefficients:

*Here and later the tilde sign pertains to the experimental function; its absence corresponds to the model description of this function.

$$l_i = \alpha \int_0^\infty \varkappa(t_1 + d_0)\, \Psi_i(t_1)\, dt_1,$$

neglecting the behavior of the function $\varkappa(t)$ in the "dead" time segment $[0, d_0)$. We confine ourselves to the first two terms in the Taylor series expansion of the function $\varkappa(t)$ and use Laplace-transform theorems on the finite value and on the integration of the inverse transform:

$$\frac{1}{\alpha}\, l_i = \lim_{s\to 0} s\left[\frac{1}{s} V_i(s)\right] = \lim_{s\to 0} V_i(s), \tag{5}$$

where

$$V_i(s) \bullet\!\!-\!\!\circ\, v_i(t_1) = [\varkappa(t_1) + d_0 \varkappa'(t_1)]\, \Psi_i(t_1). \tag{6}$$

Let i = 0. Then

$$\frac{1}{\alpha}\, l_0 = \lim_{s\to 0} V_0(s). \tag{7}$$

To seek the transform $V_0(s)$ we apply the theorems on damping and on the differentiation of the inverse transform:

$$V_0(s) = \sqrt{a}\left[\mathrm{K}\left(s+\frac{a}{2}\right) + d_0\left(s+\frac{a}{2}\right)\mathrm{K}\left(s+\frac{a}{2}\right)\right] = \sqrt{a}\left[1 + d_0\left(s+\frac{a}{2}\right)\right]\mathrm{K}\left(s+\frac{a}{2}\right). \tag{8}$$

Designating

$$\mathrm{K}_0 = \lim_{s\to 0} \mathrm{K}\left(s+\frac{a}{2}\right), \quad \mathrm{K}_{01}' = \lim_{\substack{s\to 0\\ a=1}} \mathrm{K}\left(s+\frac{a}{2}\right)$$

and substituting Eq. (8) into (7), we obtain

$$\frac{1}{\alpha}\, l_0 = \sqrt{a}\,\mathrm{K}_0\left(1 + \frac{d_0}{2}\right). \tag{9}$$

Let i = 1. To calculate $V_1(s)$ we use the theorems indicated above and the theorem on the differentiation of the transform:

$$v_1(t_1) = [\varkappa(t_1) + d_0\varkappa'(t_1)]\sqrt{a}\,(1 - at_1)\exp\left(-\frac{a}{2}t_1\right) = v_0(t_1) - ad_0t_1\varkappa(t_1)\,v_0(t_1) \circ\!\!-\!\!\bullet V_1(s) = V_0(s) + a\Big\{\mathrm{K}\left(s+\frac{a}{2}\right) +$$
$$+ d_0\left[\left(s+\frac{a}{2}\right)\mathrm{K}\left(s+\frac{a}{2}\right)\right]'\Big\} = V_0(s) + \sqrt{a}\,\mathrm{K}\left(s+\frac{a}{2}\right)\left\{d_0 + \left[1 + d_0\left(s+\frac{a}{2}\right)\right]\Delta_{\mathrm{K}}\right\}. \tag{10}$$

The second term in braces is close in structure to $V_0(s)$: The only complication is introduced by the auxiliary factor

$$\Delta_{\mathrm{K}} = \frac{\mathrm{K}'\left(s+\frac{a}{2}\right)}{\mathrm{K}\left(s+\frac{a}{2}\right)}.$$

To facilitate the analysis we introduce the following functions in accordance with Eq. (2b):

$$\overline{Q} \equiv \sqrt{Q} \equiv \sqrt{\mathrm{P}\left(s + \frac{\mathrm{P}}{4} + \frac{a}{2}\right)}, \quad Z \equiv \sqrt{Q}\,\mathrm{ch}\sqrt{Q} + B\,\mathrm{sh}\sqrt{Q}.$$

To shorten the notation we designate $Z' = dZ/d\overline{Q}$ and $B = P/2$. In this case the derivatives (with respect to the complex variable s) of the functions introduced above are written in the form

$$\overline{Q}' = \frac{\mathrm{P}}{2\sqrt{Q}},$$
$$Z'(s) = Z'\overline{Q}' = \overline{Q}'\,[(B+1)\,\mathrm{ch}\sqrt{Q} + \sqrt{Q}\,\mathrm{sh}\sqrt{Q}],$$

so that

$$\Delta_K = \bar{Q}' \frac{\frac{Z - \sqrt{\bar{Q}}Z'}{Z^2}}{\sqrt{\bar{Q}}} = \frac{\bar{Q}'}{\sqrt{\bar{Q}}}[1 - \sqrt{\bar{Q}}\Delta_{Zs}],$$

where $\Delta_{Zs} = Z'/Z$.

For the successive determination of $V_0'(s)$, $V_1(s)$, and the coefficient l_i from Eqs. (10), (9), and (5) it is necessary first of all to establish the limit of Δ_{Zs} as $s \to 0$:

$$\Delta_Z = \lim_{s\to 0}\frac{Z'}{Z} = \frac{(B+1)\,\mathrm{ch}\sqrt{B(B+a)} + \sqrt{B(B+a)}\,\mathrm{sh}\sqrt{B(B+a)}}{\sqrt{B(B+a)}\,\mathrm{ch}\sqrt{B(B+a)} + B\,\mathrm{sh}\sqrt{B(B+a)}}. \tag{11}$$

In Eq. (11) one can avoid awkward transcendental functions in future transformations by setting $a = 1$. Then

$$\Delta_{Z1} \equiv \Delta_Z|_{a=1} = \sqrt{\frac{B+1}{B}}.$$

With such a time scale we have

$$\Delta_{K1} \equiv \Delta_K|_{a=1} = -\frac{B}{B+1}, \quad V_{01} \equiv V_0(s)|_{a=1} = K_{01}\left(1 + \frac{d_0}{2}\right),$$

$$V_{11} \equiv V_{01}\left(1 + \frac{d_0}{1 + \frac{d_0}{2}} - \frac{B}{B+1}\right).$$

Substituting the quantity V_{01} into Eq. (7) and $V_{i1} = V_{11}$ with $i = 1$ into (5) we obtain the unknown coefficients of the expansion of the impulse function for the model:

$$l_{01} = l_0|_{a=1} = \alpha\left(1 + \frac{d_0}{2}\right)K_{01}, \tag{12}$$

$$l_{11} = l_1|_{a=1} = \alpha\left(1 + \delta - \frac{B}{B+1}\right)K_{01}, \tag{13}$$

where $\delta = d_0/(1 + d_0/2)$. With the adopted time scale the ratio $\sigma_1 = l_{11}/l_{01}$ of these coefficients does not contain transcendental functions and is suitable for the search for parameters of the model. If the diffusional model adequately describes the diffusion of a tracer in a channel, then

$$l_{01} = \tilde{l}_0|_{a=1}, \quad l_{11} = \tilde{l}_1|_{a=1}. \tag{14}$$

Then

$$\tilde{\sigma}_1 = \sigma_1 = 1 + \delta - \frac{B}{B+1}. \tag{15}$$

Hence,

$$P = 2\left(\frac{1}{\tilde{\sigma}_1 - \delta} - 1\right). \tag{16}$$

We note that $\delta \leq \tilde{\sigma}_1$, with the case of $\delta = \tilde{\sigma}_1$ corresponding to a regime of ideal stream displacement.

On the basis of the correspondence between the transforms and inverse transforms used in the analysis we were confined to two terms in the expansion of the function $\varkappa(t)$. In fact, only Ψ_0 and Ψ_1 from the family of Laguerre functions were needed in the derivation of Eq. (16); accordingly, the first two terms in Eq. (6) were sufficient to the Taylor series expansion of $\varkappa(t_1 + d_0)$. We note that the inaccuracy in the description of the phenomenon of mixing in an actual subject and the incompleteness of the representation of the impulse function for the model itself by a family of orthogonal Laguerre functions are reflected less in the smoothness of sections of the continuous curves being compared than in other geometrical characteristics, including the curvature.

Equation (16) was tested in a calculation of the effective Peclet number P for a combined flowing system consisting of a zone of ideal displacement followed by a finite number of sections of ideal mixing of equal volumes. Let $d_0 = 1/3$ for this model system and let the number of sections of ideal mixing equal three [26]. For the adopted model (1) the value of the Peclet number found [26] using the finite-difference approximation by the method of least squares is P = 13.24. Calculations made for the same model system by the method proposed in the present work give $\sigma_1 = 2/5$, $\delta = 2/7$, and P = 15.5.

One can ascertain that owing to the simplicity of Eq. (16) the limiting stage in the numerical estimation of the Peclet number P is the finding of the first two coefficients $\tilde{p}_i$ and $\tilde{q}_i$. From the point of view of the computation technique it is better not to take the improper integrals in the scale of the dimensionless time t and in Gauss–Laguerre quadratures. The point is that impulse signals have a finite duration, which becomes relatively small for large Θ, so that in working in the interval $[0, \infty)$ the integrand is reduced to zero at a series of nodes. Moreover, the expression inside the integral already includes the factor $\exp(-{}^1\!/_2 t)$; the latter assures good convergence of the improper integrals, although slower than does the factor $\exp(-t)$.

In the case of unimodal signals it is more desirable to use a Gauss–Legendre quadrature in the segment $[-1, 1]$ with a unit weight factor [1]. This requires the change of variables $\tau = r_\tau(1 + \zeta)$ so as to inscribe the range of integration $2r_\tau$ within the limits of a unit circle. The true half-duration r_τ of the impulse is determined from the condition of matching the far right node ξ_n with that time τ_W when the signal can still differ from zero: $y_W > 0$ $(y_{W+1} = 0)$. It is easy to trace the sequence of further arguments and transformations on the example of the "input" signal. The first coefficient is

$$\tilde{p}_0 = \int\limits_0^{2r_\tau/\Theta} y_1(t)\exp\left(-\frac{t}{2}\right)dt = \frac{1}{\Theta}\int\limits_0^{2r_\tau} y_1\left(\frac{\tau}{\Theta}\right)\exp\left(-\frac{\tau}{2\Theta}\right)d\tau =$$

$$= \frac{r_\tau}{\Theta}\int\limits_{-1}^{1} y_1\{\zeta[\tau(t)]\}\exp\left[-\frac{r_\tau}{2\Theta}(1+\zeta)\right]d\zeta \approx \sum_{j=1}^{2n}\Phi(\xi_j),$$

where

$$\Phi(\xi_j) = h(\xi_j)G(\xi_j); \qquad h(\zeta) = y_1(\tau)\exp\left[-\frac{r_\tau}{2\Theta}(1+\zeta)\right];$$

$$G(\xi_j) = \frac{r_\tau}{\Theta}A(\xi_j).$$

The nodes ξ_j and the weights $A(\xi_j)$ connected with them have been tabulated [27].

We seek the next coefficient in a similar way:

$$\tilde{p}_1 = \int\limits_0^{2r_\tau/\Theta} y_1(t)(1-t)\exp\left(-\frac{t}{2}\right)dt = \frac{r_\tau}{\Theta}\int\limits_{-1}^{1} h_1(\zeta)\exp(-\zeta)\,d\zeta \approx \sum_{j=1}^{2n}\Phi(\xi_j)\left[1-\frac{r_\tau}{\Theta}(1+\xi_j)\right],$$

where

$$h_1(\zeta) = h(\zeta)\left[1-\frac{r_\tau}{\Theta}(1+\zeta)\right].$$

The numerical determination of the coefficients $\tilde{q}_0$ and $\tilde{q}_1$ for the "output" signal $y_2(t_1)$ is carried out in the same sequence (allowing, of course, for the corresponding half-duration r_τ of the output signal). This variable-time version permits the use of all the nodes of the Gauss–Legendre quadrature regardless of the duration of the signal. Since the efficiency of this quadrature is usually illustrated [1, 27] on the example of monotonically varying functions, it is desirable to integrate the unimodal signals separately in two segments: up to the point τ_m of the maximum with $r_\tau = \tau_m/2$ and after it with $r_\tau = (\tau_W - \tau_m)/(1 + \xi_n)$, and to sum the results of the numerical integration.

The use of the method of Laguerre functions allows one to obtain the integral estimate (16) for the Peclet number P without neglecting in the analysis of the experimental data either the allowance for the properties of the flowing-system–measuring-instrument complex [28] or the a priori information on the order of magnitude of the estimated quantity in using the numerical statistical characteristics of the impulse function of the system [29]. Such an approach is advantageously distinguished by a considerable economy of computer time. To further refine the estimate of P and to test the adequacy of the model it is desirable to use the method of least squares with a quasi-Newtonian method of minimizing the discrepancy between the model and the real system [20]. In this case the estimate obtained by the method of Laguerre functions (close to the method of weighted moments) will be a good initial approximation.

Thus, the suggested approach allows one to increase the accuracy in the determination of the longitudinal mixing parameter of a confined system (in contrast to the method of statistical moments) and to considerably simplify the process of its calculation. In this case it is enough to have available only two Laguerre functions for the subjects under consideration and to take the time scale of these functions as equal to unity.

NOTATION

a	is the time scale factor of Laguerre functions;
$A(\xi_j)$	are the weights of Gauss–Legendre quadrature;
C	is the true concentration;
c	is the dimensionless concentration;
C_0	is the reference concentration;
d_0	is the relative delay of output signals;
D'_+	is the subspace of distributions of finite order;
e(t)	is the impulse function for the system of equations (1);
E(1, s)	is the transfer function for the same system;
E_L	is the coefficient of longitudinal diffusion;
h_1 and h_2	are the functions for computation of coefficients p_0 and q_0 and coefficients p_1 and q_1, respectively;
j	is the ordinal number of node in Gauss–Legendre quadrature;
L	is the length of working section of channel;
l, p, and q	are the coefficients of Laguerre expansions of functions $e(t_1)$, $y_1(t)$, and $y_2(t_1)$, respectively;
N	is the number of terms in Laguerre expansions of the signals;
n	is the number of nodes in Gauss–Legendre quadrature;
P	is the Peclet number;
r_τ	is the half-duration of signal;
s	is the complex variable;
t and t_1	are the dimensionless time for signals at input (y_1) and output (y_2 and e);
t_m	is the time of appearance of maximum point of signal;
u	is the velocity of liquid in channel;
x	is the dimensionless coordinate;
$y_1(t)$ and $y_2(t_1)$	are the functions describing an arbitrary input signal (x = 0) and the response to it (x = 1);
z	is the true coordinate;
α	is the constant multiplier of transfer function;
Δ	are the logarithmic derivatives of complex expressions;
ξ	is the integration variable in the interval [−1, 1];
Θ	is the average time of stay of liquid in channel;
K(s) and $\varkappa(t)$	are the variable multiplier of transfer function and its Laplace inverse transform;
ξ_j,	is the j-th node in Gauss–Legendre quadrature;
σ_1	is the ratio of coefficients of Laguerre series expansion of impulse function;
τ	is the real time;
τ_d	is the real signal delay time;
τ_m	is the real time of appearance of signal maximum;
Ψ_i	is the Laguerre function of i-th order.

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NUMERICAL STUDY OF LAMINAR SWIRLED FLOW IN AN ANNULAR CHANNEL

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The effect of stream rotation on the velocity distribution in an annular channel is studied by the numerical method. The degrees of swirling corresponding to the initiation of stream separation are presented as functions of the Reynolds number for different values of the geometrical parameter of the channel.

I. It is known that the intensity of processes of heat and mass transfer in annular channels and pipes increases when swirled flows are used in them [1].

The attempts at an analytical solution of such problems are connected with certain simplifying assumptions. For example, the problem of the development of Poiseuille flow in a straight round pipe with stream rotation was solved in [2]. It was assumed that the changes in the flow caused by this rotation are small. This allowed the authors to solve the problem in a linear formulation. An approximate calculation of the development of swirled flow of a viscous incompressible liquid in a cylindrical pipe was the subject of [3], where assumptions were made that the radial velocity component and its derivative with respect to the radius are small, as well as the assumption that the axial velocity component differs little from its average value over the cross section.

Another approach to the solution of such problems is the numerical integration of the equations of motion of a viscous liquid. The velocity profiles of swirled flow in a round pipe were calculated in [4] using the method of [5]. It was found that the assumptions of [2] are not always satisfied.

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